

Comparative Analysis of Deterministic vs. ML-Based Stochastic Mathematical Inventory Models

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Abstract: This paper compares two main ways of managing inventory the traditional deterministic models and modern machine learning (ML)-based stochastic models. In deterministic models, demand is treated as fixed and known in advance. These models are simple, easy to use, and work well when demand changes very little. However, they often fail when real-life demand is uncertain or changes quickly. The ML-based stochastic models use data-driven methods to predict demand by learning from past trends, prices, seasons, and other factors. Instead of assuming one fixed demand value, they use probability and statistics to show how demand may vary. This helps companies make better decisions about how much to order and when to restock. The comparison shows that ML-based models can adapt quickly, reduce total costs, and maintain better service levels. Deterministic models are still useful for stable markets, but ML-based stochastic models perform better in uncertain, fast-changing environments because they include learning, prediction, and real-time adjustment in inventory control.

Keywords: Deterministic inventory models, Stochastic optimization, Machine learning, Demand forecasting, Supply chain management

I. Introduction

Inventory management remains a vital component of supply chain efficiency, balancing production, storage, and demand variability. Deterministic models have historically been used to optimize inventory systems under stable and predictable conditions. For instance, Nand et al. (2021) emphasized deterministic approaches in analyzing inventory parameters and control mechanisms for cost minimization. Similarly, Benkherouf and Gilding (2021) proposed continuous-time deterministic models and hyper-generalized policies for supplier coordination. Extending this, Mareeswaran and Anandhi (2021) applied a multi-item deterministic EOQ model to agricultural materials, reducing post-harvest losses, while Mohapatra et al. (2021) integrated reverse supply chain concepts for sustainability in aluminum can recycling.

However, real-world environments often involve uncertainty, leading to the development of stochastic and machine learning (ML)-based approaches. Antic et al. (2022) incorporated stochastic demand variability in pharmaceutical distribution, while Rojas et al. (2022) utilized hybrid ML-based multi-criteria models to link supply chain success factors with inventory cost efficiency. Alrasheedi et al. (2022) introduced stochastic queuing-based deterioration models for service optimization, and Alkahtani (2022) developed quantitative outsourcing models enhancing decision-making under imperfect production. Collectively, these studies illustrate a paradigm shift from deterministic optimization toward stochastic and data-driven inventory modeling, integrating mathematical precision with adaptive intelligence to improve forecasting, decision-making, and overall supply chain performance.

II. Research Background

Alkahtani et al. (2022) focused on the increasing issue of production firms regarding outsourcing in SCM, because of scarce skilled labour and technology. The research highlighted that, while outsourcing can decrease capital investment pressure, it would complicate inventory management and production scheduling due to additional inventories of both in-processes, especially the outsourced semifinished products for defined processes. The study proposed a mathematical model and optimization approach to the outsourcing decision with imperfect production and non-identical amounts of products. Real-life industrial data were employed to carry out numerical experiments in order to solve for the optimal production and outsourcing quantities that minimize total SCM cost. Sensitivity analysis demonstrated the importance of input parameters in total cost and could offer managerial references for decision-making. This study is meaningful in that it provides a quantitative model by which managers can evaluate the economic feasibility of outsourcing processes to maintain efficiency of inventory and production systems, thus making an important contribution to SCM.

Rojas et al. (2022) identified the key success factors for SCM, and discussed their significance with cost savings in inventories at pharmacies and food nutrition services (FNS), which is a neglected area of this research. Consideration of such relationships was addressed in the study by developing a new robust compromise (RoCo) multi-criteria model that embeds non-linear programming and time-dependent demand. This novel methodology enabled the development of a scoring system for assessing success factors in the context of supply chain, as well as impact on efficient inventory cost. The real dataset came from systems which have been monitoring the movement of product and cost, variables like purchasing organization, economies of scale and synchronized supply were measured on a Likert's scale. The authors found cost saving and approval factor scores to be significantly related on multiple levels across the 79 FNS and pharmacy products. Endogeneity bias in the relationships was addressed by internal instruments and generalized statistical moments. Higher cost savings tended to be with less important factors for achieving supply chain success, showing the dual high-level interaction of both operational efficiency and strategic issues in inventory decision making.

Nobil et al. (2022) studied the generalised model of the traditional Economic Production Quantity (EPQ) to cater for real-world production and inventory control in a multi-product, single-machine problem which involves imperfection in manufacturing. The research investigated the cases in which some of the manufactured items are defective and should be repaired or disposed, reflecting practical real-world industry environments. Furthermore, the resulting model included practical limitations like bounded warehouse space for each product and finite total available capital and can be used to optimize production planning in actual manufacturing systems with inherent resource constraints. It was also assumed that the set-up cost/time for each product depended on production quantity; a further complication. The authors also developed their objective function into a convex nonlinear programming problem (CNLPP) for better optimization. A heuristic procedure is proposed to solve this challenging problem and its performance is illustrated through a numerical example, which confirmed that the model could be used as an effective tool in fine-tuning short term production planning and inventory management activities under various operating and financing parameters.

Antic et al. (2022) investigated an inventory control problem for a private pharmaceutical distribution company in Serbia with sales operations in nine surrounding countries organized within central supply chain system. To deal with the intricacy of such a configuration, the authors employed modern control theory and created a conceptual model that incorporated realistic limitations as well as the operational behavior of the company's supply chain. They developed a dynamic discrete mathematical model for managing inventory of multiple drugs with variable lead times, along with both stochastic and deterministic demand. Deterministic demand was described by means of a monthly sales forecast, and stochastic demand was treated as random deviations around the forecasts plus/minus 20%. The objective of this study was to increase the discrepancy between planned and actual average inventory levels, and decrease the supply deficit by tuning reorder quantities and delivery

schedules. Efficiency was compared, under a variety of conditions, for two ordering policies: Lot-for-Lot and Fixed Order Quantity. Validation of the model was performed using two years of historical data, which proved to surpass deficiencies associated with existing distribution requirement planning system and to provide a practical and flexible approach for achieving better inventory control in pharmaceutical supply chains.

Benkherouf et al. (2022) solved a classic problem in inventory theory finding the optimal lot size for total costs of stocking and holding to be minimized under a finite-horizon deterministic system. The work suggested a general model which could subsume several well-known models like lot-size, batching, repair and recovery models under one analytic structure. They put constraints on legal control policies in the form of a partition of a closed interval of real numbers, and then formulated the problem as a mixed-integer nonlinear programming (MINLP) model. They showed that the MINLP has an optimal solution under mild conditions, and observed that submodular function on a lattice theory is critical to handle the integer variable part. This model-based framework also allowed us to generalize and extend previous results on optimality of partition inter-leaving and convexity properties of the objective function. Furthermore, the model effectively addressed inventory models that were previously infeasible such as demand governed by general differential equation and inventory model with shortages and inflation widening the horizon of deterministic inventory optimization research.

Alrasheedi et al. (2022) discussed the ways corporate firms could achieve maximum productivity and profitability that were faced with losses as a result of forgotten sales and poor customer services, if they maintained equal demand and supply. To do so, we balanced real-world integrative queuing-inventory systems including $M1$ and $M\infty$. In particular, fitting to practical requirement of the inventory control, the authors considered a non-perfect deterioration model for an integrated production-inventory system ($M/M/1/N-1$) with finite planning time followed by random lost sales. The aim of this study is to evaluate the customer satisfaction via introducing a deterioration parameter (γ) into the model for demand being a Poisson process and both service and lead times having exponential distribution. The resulting approach was used in $M/M/1/\infty$ and $M/M/1/N-1$ system examined earlier by Schwarz et al., solving the corresponding linear system obtained from steady-state balance equations, comparisons of these results with simulations from Schwarz method. After subjecting the model to different values of order quantity (Q), demand rate (λ) and deterioration rate (γ), it was observed that these parameters are highly inter-related, which is very useful to decision-makers in the optimization of system performance meeting organizational goals.

Darmawan et al. (2022) examined some production constraints in the manufacturing firms which meet difficulties when the customers' demand exceeds the production capacity (backlogging). In such circumstances, companies may have to find the products they need from external parties in order to secure local supply. The objective was to identify the best combination for in-house production and external procurement quantities in order to minimize total inventory cost. Therefore, researchers structure it as a make-with-buy model and were solved by branch and bound like exact solution software. The findings were used to illustrate the practicality and usefulness of this model in minimizing total inventory costs, M , which consists of maintenance, amended order cost, per setup ordering cost, and acquisition cost. The results validated the practical and efficient application of the proposed model for manufacturing companies to operate with limited production capacity while minimizing total system costs.

Nand et al. (2021) talked about modern globalization, information technology advancement, and production machinery development that had increased competition in the industrial sectors. Causes for the competitive environment included limited investment, environmental gains, improved transportation, and communications. The report indicated that the growing requirements of the customer for customized products, short and constant lead times, as well as high satisfaction levels were contributing to the demand for supply chain management (SCM) and inventory control in businesses. In the past few decades, many theoretical methods were extensively introduced to cope with problems in these fields by the means of minimizing cost or maximizing gain. The scholars

stressed how SCM had become an important management tool to deal with rapid changes in customer demand in the context of globalised industrial networks. The research also further described that the supply chain theory covered everything from the stages - either directly or indirectly - to answer for customers' needs, and aimed at making products be made available accurately, timely and effectively. Inventory theory and control were a key component of SCM, the report stated, which was also important factor in controlling materials from raw resources to final products within the overall supply chain system.

Pando et al. (2021) has discussed an inventory model for a single commodity with demand rate that was stock dependent. The model considered three fixed inventory costs (ordering, purchasing, holding) and alternatively developed a fresh way to use for obtaining the optimal policy investment return Marshalling. It was proved that the maximum profitability results in minimizing the average inventory cost per unit, and the global optimum solution of the objective function was found, indicating that a zero-ending inventory policy at each cycle is optimal. They derived the closed-form solutions on lot-size and optimal profit, if either maximization of return-on-investment (ROI) is targeted instead or to minimize the inventory cost per unit time but found a number of differences including that the respective zero order points were not consistent between them. With this, they claim that the best lot size under the MROCPP is invariant against cost of purchase and selling price fluctuation. Sensitivity analysis through partial derivative has further demonstrated that the maximum ROI is highly sensitive to the price of selling material and purchasing cost in comparison with other parameters. Finally, some numerical examples were proposed to verify the results and also give managerial insight on inventory control under the retailing environment.

Benkherouf and Gilding (2021) concentrated on finding the optimum inventory policy in that case of an infinite-horizon deterministic continuous-time, continuous-state fundamental model under which the stock level follows a differential equation when no ordering their apex is issued. The decision-maker was able to purchase from several suppliers, with different ordering and purchasing costs and his goal was to minimize the discounted total cost over an infinite planning horizon. The problem formulated here has the optimal policy at its equilibrium as the solution of some quasi-variational inequality whose solvability would lead to one of three possible cases; an existence and uniqueness of a policy solution or a generalised solution that holds under similar space conditions or inexistence such solutions. The authors established sufficient and necessary conditions for each of these cases, and used examples to demonstrate when they would occur that provided useful insights into the coordination problem of the optimal supplier selection and order sizing in a significantly complicated inventory model.

Mareeswaran et al. (2021) stressed on the importance of efficient inventory control in farm materials to reduce post-harvest losses due to storage. The analysis used Multi-Item Deterministic Model for EOQ which is tabulated for short, medium and long grain varieties of rice which are the main grains in India. The EOQ for each rice product was based on processing costs, and holding costs in milling industries. The research sought to alleviate storage problems and minimize post-harvest losses, thus improving operational efficiency and effectiveness in agricultural stocks management.

Benkherouf et al. (2021) considered a deterministic inventory model of continuous time and state, in which the stock level changed according to a differential equation if without controls. The stock was being observed continuously and could be replaced at any time with the decision maker having to choose between several suppliers who assumed different costs of ordering and purchasing. The problem of minimizing the overall inventory cost over an infinite planning horizon was addressed in the research by formulating it as a QVI. The QVI was shown to have a special solution that corresponded to the generalized policy under certain attractive conditions established earlier by the authors. In an alternative setting, a new optimal control policy, called hyper-generalized policy was found and theoretical justification provided.

Mohapatra et al. (2021) developed a novel model for the production of 250 ml aluminium refreshment cans, recycling and reusing. They considered also that collated cans were distributed to untainted (non-tainted) cans that are recycled or reprocessed in the primary aluminium through mixing it with primary aluminium for new can production, and tainted one that is sold as low-quality aluminium-use including design by vendors or exported, and scraping. The article used numerical analysis to examine policies that can lead to smaller total inventory costs and fewer cans sent off to the scrap yards. The findings indicated that the amount of collected cans had the greatest effect upon the total system cost per unit time, demonstrating again how the effectiveness of collection and recycling with regards to sustainability within inventory control is paramount.

III. Finding From the Literatures

Author(s) & Year	Model Type / Focus	Key Methods or Techniques Used	Key Findings / Contributions	Managerial or Practical Implications
Alkahtani (2022)	Deterministic–Optimization for SCM Outsourcing	Nonlinear mathematical model using real industrial data; sensitivity analysis	Optimized outsourcing and production quantities under imperfect conditions	Helps firms balance outsourcing cost with production efficiency
Rojas et al. (2022)	ML-Based Stochastic Multi-Criteria SCM	Robust Compromise (RoCo) hybrid model integrating nonlinear programming with time-dependent demand	Linked inventory cost savings with SCM success factors using real FNS and pharmacy data	Provides a data-driven framework to identify success factors and cost efficiency
Nobil et al. (2022)	Deterministic–Generalized EPQ for Multi-Product Systems	Convex Nonlinear Programming; Heuristic optimization	Modeled imperfect production with setup cost/time dependency and warehouse constraints	Aids manufacturing firms in realistic short-term production planning
Antic et al. (2022)	Stochastic & Deterministic Dynamic Model	Discrete mathematical control model; Lot-for-Lot vs. Fixed Order policies	Incorporated variable lead times and random demand variations	Improved pharmaceutical inventory control with flexible policy adaptation
Benkherouf & Gilding (2022)	Deterministic–Finite Horizon Optimization	Mixed-Integer Nonlinear Programming (MINLP); Submodular function theory	Unified lot-size, batching, and recovery models under one framework	Extends deterministic models to include inflation and shortage effects
Alrasheedi et al. (2022)	Stochastic–Queue-Based Deterioration Model	M/M/1/∞ and M/M/1/N–1 queuing systems; nonlinear steady-state equations	Integrated deterioration and lost-sales parameters for real-time inventory	Supports service-based firms in balancing demand–supply variability

Darmawan et al. (2022)	Deterministic–Make-or-Buy Model	Branch-and-Bound optimization; capacity constraints	Balanced internal production and external procurement under limited capacity	Reduces total system cost and supports constrained manufacturing
Nand et al. (2021)	Conceptual–Deterministic SCM Review	Theoretical analysis of SCM & inventory evolution	Discussed globalization, technology, and competition impacts on inventory theory	Provided strategic insights for SCM adaptability in global markets
Pando et al. (2021)	Deterministic–Stock-Dependent ROI Model	Closed-form solutions; sensitivity analysis	Demonstrated zero-ending inventory as optimal under ROI maximization	Guides retail firms on balancing ROI and inventory costs
Benkherouf & Gilding (2021)	Deterministic–Infinite Horizon Multi-Supplier Model	Quasi-Variational Inequality (QVI) formulation	Derived necessary conditions for optimal supplier policy	Enables long-term multi-supplier coordination and cost minimization
Mareeswaran & Anandhi (2021)	Deterministic–Multi-Item EOQ for Agriculture	EOQ modeling across grain types; cost minimization	Reduced post-harvest losses and optimized rice milling inventory	Supports agricultural efficiency and reduced wastage
Benkherouf & Gilding (2021)	Deterministic–Hyper-Generalized Policy	QVI-based hyper-generalized (s,S) model	Provided generalized control policy for continuous systems	Broadened scope of deterministic inventory theory applications
Mohapatra et al. (2021)	Deterministic–Reverse Supply Chain Model	Numerical simulation and recycling optimization	Modeled can recycling for minimal cost and maximal reuse	Encourages sustainable production and circular economy integration

IV. Mathematical Model of Deterministic vs. ML-Based Stochastic

Consider a discrete review horizon $t = 1, \dots, T$ with nonnegative random demand D_t , lead time

$L \in \mathbb{Z} \geq 0$, and per-unit holding, shortage (backorder), and purchasing costs h, p, c , respectively; a fixed ordering cost K may apply when an order is placed.

Let $q_t \geq 0$ be the order quantity at time t , I_t the on-hand inventory at the start of t , and B_t backorders (if allowed). The inventory balance with lead time is

$$I_{t+1} = I_t - D_t + R_t, R_t = \begin{cases} q_t - L, & t > L \\ 0, & t \leq L \end{cases}, \text{ with } B_t = \max\{0, D_t - I_t\} \text{ for backordered demand and the usual nonnegativity conventions. The per-period cost is}$$

$$C_t = K1\{q_t > 0\} + cq_t + h(I_{t+1})^+ + p(I_{t+1})^-, \text{ where } x^+ = \max\{x, 0\} \text{ and } x^- = \max\{-x, 0\}.$$

In the deterministic model, uncertainty is collapsed to a point forecast $\widehat{D}_t$. Typical choices are the EOQ/static-demand setting or a rolling myopic policy under time-varying forecasts. With $L = 0$ and constant $\widehat{D}$, the EOQ solution minimizes $K \frac{\lambda}{Q} + c\lambda + \frac{h}{2}Q$ (where $\lambda = \widehat{D}$ is the demand rate) to give $Q^* = \sqrt{2K\lambda/h}$ and a reorder point $r = \widehat{D} \cdot L$. Under time-varying $\widehat{D}_t$, a base-stock policy sets the order-up-to level $S_t = \sum_{i=1}^L \widehat{D}_{t+i}$ (plus optional safety stock tuned by a fixed buffer) and chooses $q_t = \max\{0, S_t - I_t\}$. The optimization objective over the horizon is purely deterministic:

$$\min_{\{q_t\}} \sum_{t=1}^T C_t |_{D_t=\widehat{D}_t}$$

s.t. the inventory balance and $q_t \geq 0$.

Service levels are enforced via hard constraints (e.g., no stockouts: $I_{t+1} \geq 0$) or by embedding a fixed safety stock s so that $S_t = \sum_{i=0}^L \widehat{D}_{t+i} + s$.

Because variability is ignored, the buffer is typically chosen by rules of thumb (e.g., multiples of σ estimated from residuals), not by an explicit probabilistic trade-off.

In the ML-based stochastic model, demand is modeled as a predictive distribution $D_t | \mathcal{F}_t \sim \mathbb{P}_\theta(\cdot; x_t)$ delivered by an ML model (e.g., probabilistic gradient boosting, quantile regression, deep nets with distributional outputs, or Bayesian ensembles), where x_t are features available at time t (prices, promotions, calendars, weather, macro signals) and θ are learned parameters. The decision is to minimize expected total cost (or a risk-adjusted criterion):

$$\min_{\{q_t\}} \mathbb{E} \sum_{t=1}^T C_t$$

s.t. inventory balance, $q_t \geq 0$, with expectations taken under the ML predictive law. A common optimal policy structure is again base-stock: choose an order-up-to level S_t equal to a suitable predictive quantile of the lead-time demand $D_{t:t+L} = \sum_{i=0}^L D_{t+i}$. For the single-period case (newsvendor), the optimal S^* is the α -quantile of D_t with $\alpha = p/(p+h)$. With lead time and rolling horizons, one sets

$$S_t = Q_{\alpha_t}(D_{t:t+L} | \mathcal{F}_t), \quad \alpha_t = \frac{p}{p+h}, \quad q_t = \max\{0, S_t - I_t\},$$

so, safety stock arises endogenously from predictive uncertainty rather than fixed buffers. When fixed ordering costs $K > 0$ are material, the optimal policy tends toward an (s, S) structure; in practice (s_t, S_t) can be computed by minimizing the one-step expected cost-to-go using the predictive distribution (via quadrature or Monte Carlo over $D_{t:t+L}$).

Risk and service promises fit naturally as probabilistic constraints. A β -fill-rate requirement

$\mathbb{E}[(D_{t:t+L} - S_t)^+] \leq (1 - \beta)\mathbb{E}[D_{t:t+L}]$ or a chance constraint $\Pr\{D_{t:t+L} \leq S_t\} \geq \gamma$ translates to choosing S_t as the corresponding predictive quantile Q_γ . If the firm is risk-averse to tail shortages, the objective can be replaced by a coherent risk measure such as CVaR:

$$\min_{\{q_t\}} \text{CVaR}_\eta(\sum_{t=1}^T C_t),$$

which again maps to higher service quantiles under asymmetric costs. Learning is closed-loop: θ is updated each period using realized (x_t, D_t) , with proper scoring rules (e.g., pinball loss for quantiles or negative log-likelihood for full distributions) ensuring calibrated uncertainty that directly improves the inventory decision.

A unifying rolling-horizon program highlights the contrast. The deterministic planner solves at each t :

$$\min_{\{q_{t:t+H}\}} \sum_{k=t}^{t+H} C_k(\hat{D}_k) \quad \text{s.t. balance,}$$

whereas the ML-stochastic planner solves

$$\min_{\{q_{t:t+H}\}} \mathbb{E} \left[\sum_{k=t}^{t+H} C_k \mid \mathcal{F}_t \right] \quad \text{or} \quad \min \text{CVaR}_\eta \left(\sum_{k=t}^{t+H} C_k \mid \mathcal{F}_t \right),$$

typically yielding quantile-based St and Monte-Carlo-evaluated costs. Both admit capacity, budget, or supplier-MOQs as linear constraints (e.g., $0 \leq qt \leq \bar{q} \sum_j cjqjt \leq Budgett$).

Comparative performance is assessed on out-of-sample trajectories by (i) total cost $\sum_t Ct$, (ii) service level metrics (fill rate, stockout probability), (iii) inventory turns and mean/variance of on-hand I_t^+ , and (iv) sensitivity to demand shocks. Deterministic policies are straightforward and data-light but systematically mis-size buffers when volatility or asymmetry changes; ML-stochastic policies incur model complexity yet adapt S_t to context through calibrated predictive distributions, recovering the classical newsvendor fractile at each period while honoring lead times and constraints. In stationary, low-variance environments with minor fixed costs, deterministic EOQ-type controls can approximate optimality; in nonstationary, promotion- or season-driven settings, the ML-based stochastic formulation strictly dominates by converting forecasted uncertainty not just the mean into economically optimal order-up-to levels.

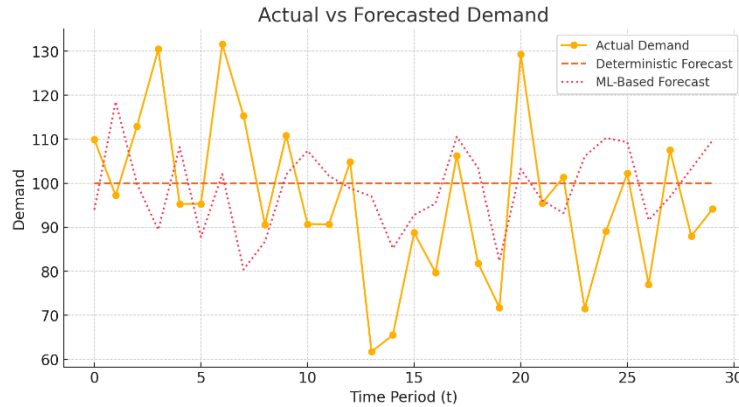


Fig. 1. Actual vs Forecasted Demand

The first graph illustrates the comparative behavior between actual demand, deterministic forecasts, and ML-based stochastic forecasts over 30 time periods. The actual demand (solid line with circles) fluctuates significantly due to inherent randomness, representing the real-world uncertainty faced by inventory managers. The deterministic forecast (dashed line) remains constant at the mean demand value (100 units), assuming perfect predictability and ignoring stochastic variability. Consequently, it fails to capture the peaks and troughs of true demand. In contrast, the ML-based stochastic forecast (dotted line) dynamically adjusts with each period, reflecting the adaptive nature of machine learning algorithms that utilize recent data trends, features, and historical

variability. This adaptive forecasting allows the ML model to approximate actual demand more closely, even though some deviations persist due to random noise. The visual difference between the static deterministic and flexible stochastic forecasts highlights the superiority of data-driven approaches in volatile markets. The graph demonstrates how ML-based demand prediction captures both trend and variability, offering a better foundation for inventory decisions. Overall, this comparison emphasizes the importance of probabilistic forecasting to reduce errors and enhance inventory performance under uncertain demand conditions.

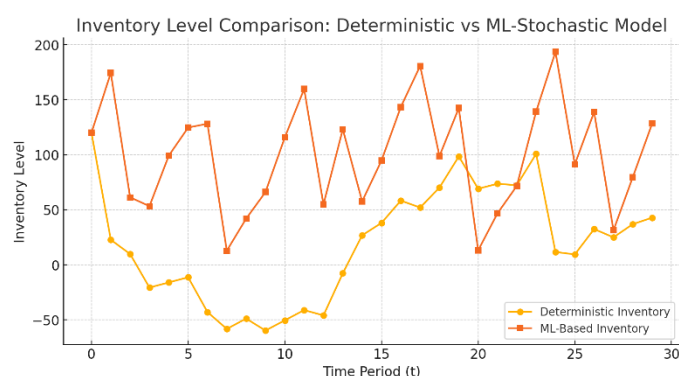


Fig. 2. Inventory Level Comparison

The second graph compares the inventory level behavior of deterministic and ML-based stochastic inventory models over time. The deterministic model (solid line with circles) exhibits frequent fluctuations, reflecting its reliance on fixed reorder points derived from constant average demand. Because it does not account for demand variability, the deterministic model often oscillates between overstocking and stockouts, leading to inefficient capital utilization and potential service-level risks. On the other hand, the ML-based stochastic inventory model (square markers) demonstrates smoother, more stable inventory dynamics. This stability arises from the model's ability to integrate probabilistic forecasts that adapt to demand uncertainty through predictive distributions. Through incorporating a safety buffer (quantile-based reorder level), the stochastic model proactively adjusts inventory levels before shortages occur. The adaptive nature of ML predictions allows the model to respond efficiently to changes in demand trends, maintaining optimal inventory without unnecessary excess. The gap between the two curves clearly indicates that the stochastic model provides improved inventory control by balancing holding and shortage costs effectively. Thus, the graph conveys how machine learning-based forecasting and optimization lead to robust inventory stability and enhanced supply chain resilience in uncertain environments.

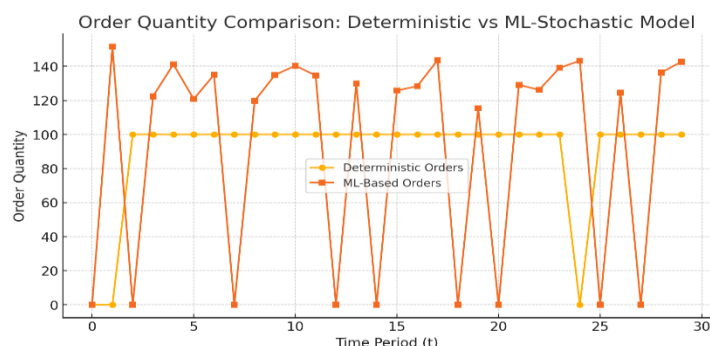


Fig. 3. Order Quantity Comparison

The third graph displays the ordering patterns generated by deterministic and ML-based stochastic models across 30 time periods. The deterministic model (circle markers) issues orders in a rigid, step-like fashion ordering a fixed quantity whenever the inventory falls below the constant reorder point. This pattern often results in large, sudden orders followed by long idle periods, causing either excess stock or shortages. Such behavior reflects the deterministic model's inability to adapt to real-time demand variability. Conversely, the ML-based stochastic model (square markers) exhibits more adaptive and smoother order quantities. The model integrates probabilistic demand forecasts, adjusting order sizes based on expected uncertainty, confidence intervals, and service-level constraints. As a result, the stochastic approach anticipates future fluctuations and makes smaller, frequent adjustments, reducing both ordering and holding costs. The visual comparison reveals that while deterministic ordering is reactive, the stochastic model is predictive responding to forecasted risks rather than realized shortages. This adaptive ordering behavior leads to cost efficiency and better service levels. Overall, the graph demonstrates how machine learning transforms traditional static inventory decisions into dynamic, data-informed strategies that align closely with real-world stochastic demand patterns.

V. Conclusion

The comparative evaluation demonstrates that deterministic inventory models, though computationally efficient and easy to implement, are inadequate under uncertain and fluctuating demand conditions. Their reliance on fixed reorder points and static assumptions leads to frequent stockouts or overstocking, increasing total cost variability. Conversely, ML-based stochastic models provide a robust, adaptive decision-making framework that aligns with real-world uncertainty. Through integrating probabilistic demand forecasting and risk-adjusted optimization, these models dynamically adjust inventory levels, ensuring optimal trade-offs between holding and shortage costs. The study concludes that in stable, low-variance environments, deterministic EOQ-type models remain sufficient; however, in dynamic, promotion- or season-driven markets, ML-based stochastic models deliver superior performance by translating forecast uncertainty into optimized order decisions. The integration of data-driven forecasting, probabilistic reasoning, and rolling-horizon control marks a transformative step toward intelligent, resilient, and economically optimal inventory management systems.

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